Final Exam Review 12/4/2023

SI Final Exam Review Session Thu Dec 7 at 5:30pm in UNIV 019

Posted Nov 28, 2023 4:19 PM

Supplemental Instruction will be holding a review session for the Final Exam.

SI Final Exam Review Session

Date: Thursday, December 7, 2023

Time: 5:30pm – 7:20pm

Location: UNIV 019

Reminder: There are also regularly scheduled SI sessions and office hours that are held throughout the week. This information can be found under the Fall 2023 Supplemental Instruction course on Brightspace.

Final Exam

Posted Nov 14, 2023 2:06 PM

The final exam will be on Friday, December 15 from 10:30am-12:30pm.

A memo with detailed information will be posted in Brightspace closer to the exam.

Plan travel arrangements accordingly. No final exams will be given early.

Lesson 29 #4.

Let the function

$$g(x,y) = -\frac{1}{2}x^2 + xy - \frac{1}{25}y^4 - 5,$$

whose first order partial derivatives are

$$g_x = -x + y \quad \text{and} \quad g_y = x - \frac{4}{25}y^3.$$

Determine the critical points and show how many of these points are relative maxima, relative minima, and saddle points. (Answer from the smallest x-values to the largest).

Formula Sheet

Second Derivative Test

Given the critical point (a, b), such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$, and let

$$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - [f_{xy}(a,b)]^2$$

- If $D > 0$ and $f_{xx}(a,b) > 0$ then $f(a,b)$ is a relative minimum.
- If $D > 0$ and $f_{xx}(a,b) < 0$, then $f(a,b)$ is a relative maximum.
- If $D < 0$, then $f(a,b)$ is a saddle point.

$g_x = -x + y$ $\qquad$ $g_y = x - \frac{4}{25}y^3$

$$\begin{cases} -x + y = 0 & (1) \\ x - \frac{4}{25}y^3 = 0 & (2) \end{cases}$$

(1) $y = x \rightarrow (2)$ $\quad y - \frac{4}{25}y^3 = 0$

$y\left(1 - \frac{4}{25}y^2\right) = 0$

$y = 0$

$y = \pm\frac{5}{2}$

Because $x = y$

$(0,0)$

$\left(-\frac{5}{2}, -\frac{5}{2}\right)$

$\left(\frac{5}{2}, \frac{5}{2}\right)$

$g_x = -x + y$ $\qquad$ $g_y = x - \frac{4}{25}y^3$

$g_{xx} = -1$

$g_{xy} = 1$ $\qquad$ $g_{yy} = -\frac{12}{25}y^2$

$D(x,y) = (-1)\left(-\frac{12}{25}y^2\right) - [1]^2 \quad = \frac{12}{25}y^2 - 1$

$D(0,0) = -1 < 0$ saddle point

$D\left(-\frac{5}{2}, -\frac{5}{2}\right) = \frac{12}{25}\left(\frac{25}{4}\right) - 1 = 3 - 1 = 2 > 0$

$g_{xx}\left(-\frac{5}{2}, -\frac{5}{2}\right) = -1 < 0$

local max

$D\left(\frac{5}{2}, \frac{5}{2}\right) = \frac{12}{25}\left(\frac{25}{4}\right) - 1 = 3 - 1 = 2 > 0$

$g_{xx}\left(\frac{5}{2}, \frac{5}{2}\right) = -1 < 0$ local max

Lesson 28 #8

Compute the second order partial derivatives of

$$f(x,y) = 3y \cdot e^{\cos(5x-4)}$$

$f_x(x,y) = 3y\, e^{\cos(5x-4)} \cdot (-\sin(5x-4)) \cdot 5$

$f_x(x,y) = -15y\, e^{\cos(5x-4)} \sin(5x-4)$

$f_y(x,y) = 3e^{\cos(5x-4)}$

$f_{xx}(x,y) = -15y\, e^{\cos(5x-4)} \cos(5x-4) \cdot 5 + (-15y) e^{\cos(5x-4)} (-\sin(5x-4))(5) \sin(5x-4)$

$= -75y\, e^{\cos(5x-4)} \cos(5x-4) + 75y\, e^{\cos(5x-4)} (\sin(5x-4))^2$

$= 75y\, e^{\cos(5x-4)} \left(-\cos(5x-4) + (\sin(5x-4))^2\right)$

$f_{yy}(x,y) = 0$

$f_{xy}(x,y) = f_{yx}(x,y) = 3e^{\cos(5x-4)} \cdot (-\sin(5x-4)) \cdot 5 = -15e^{\cos(5x-4)} \sin(5x-4)$

Lesson 35 #6.

Practice Problem

Submissions are not permanently recorded

A large building with a rectangular base has a curved roof whose height is

$$h(x,y) = 84 - 0.04x^2 + 0.027y^2$$

The rectangular base extends from $-50 \le x \le 50$ feet and $-100 \le y \le 100$ feet.

Find the average height of the building, round your answers to the nearest 3 decimal places.

Area of rectangle = 100 × 200 = 20,000

$$\text{Ave} = \frac{\int_{-50}^{50}\int_{-100}^{100} (84 - 0.04x^2 + 0.027y^2)\,dy\,dx}{20{,}000}$$

$$= \frac{1}{20{,}000}\int_{-50}^{50}\left[84y - 0.04x^2y + 0.027\frac{y^3}{3}\Big|_{y=-100}^{y=100}\right]dx$$

$$= \frac{1}{20{,}000}\int_{-50}^{50}\left[(8400 - 4x^2 + 9000) - (-8400 + 4x^2 - 9000)\right]dx$$

$$= \frac{1}{20{,}000}\int_{-50}^{50}(34{,}800 - 8x^2)\,dx$$

$$= \frac{1}{20{,}000}\left[34{,}800x - \frac{8x^3}{3}\Big|_{-50}^{50}\right]$$

$$= \frac{1}{20{,}000}\left[\left(1{,}740{,}000 - \frac{1{,}000{,}000}{3}\right) - \left(-1{,}740{,}000 + \frac{1{,}000{,}000}{3}\right)\right]$$

≈ 140.666667